IAC 2D Vector Review Sheet 2011 Answer Key is vectorious!

Calculator Allowed

Show work. Round your answers to the *thousandths* place when applicable.

1. Show that the vector from R = (-4, 2) to S = (-1, 6) is equal to the vector from P = (2, -1) to Q = (5, 3).

$\vec{RS} = \langle -1-{-4}, 6-2 \rangle = \langle 3, 4 \rangle$ $\vec{PQ} = \langle 5-2, 3-{-1} \rangle = \langle 3, 4 \rangle$

2. Vector **v** has initial point (-3, 4) and terminal point (-5, 2). Find |**v**|.

$\vec{v} = \langle -5-{-3}, 2-4 \rangle = \langle -2, -2 \rangle$ $|\vec{v}| = \sqrt{(-2)^2+(-2)^2} = \sqrt{8} = \boxed{2\sqrt{2}}$

3. Given **v** = ⟨-4, -2⟩ and **u** = ⟨-8, 6⟩, find:

a. 2(**v** + **u**)

$2\langle -12, 4 \rangle = \boxed{\langle -24, 8 \rangle}$

b. |2**v** + 2**u**|

$\sqrt{(-24)^2+8^2} = \sqrt{640} = \boxed{8\sqrt{10}}$

c. (7/6)**v** – (2/3)**u**

$\frac{7}{6}\langle -4, -2 \rangle - \frac{2}{3}\langle -8, 6 \rangle = \langle -\frac{14}{3}, -\frac{7}{3} \rangle + \langle \frac{16}{3}, -\frac{12}{3} \rangle = \boxed{\langle \frac{2}{3}, -\frac{19}{3} \rangle}$

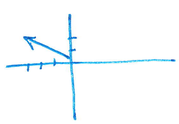
4. Find a unit vector in the direction of **v** = ⟨-4, -5⟩.

$|\vec{v}| = \sqrt{(-4)^2+(-5)^2} = \sqrt{16+25} = \sqrt{41}$

$\boxed{\langle -\frac{4}{\sqrt{41}}, \frac{-5}{\sqrt{41}} \rangle}$

5. Find the direction angle (in degrees) of each vector.

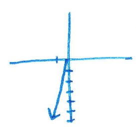
a. **u** = ⟨-3,2⟩



$\tan\theta = -\frac{2}{3}$

$\theta = -33.690 + 180 = \boxed{146.310°}$

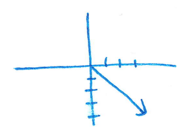
b. **v** = ⟨-1, -6⟩



$\tan\theta = 6$

$\theta = 80.538 + 180 = \boxed{260.538°}$

c. **w** = 3**i** - 4**j**

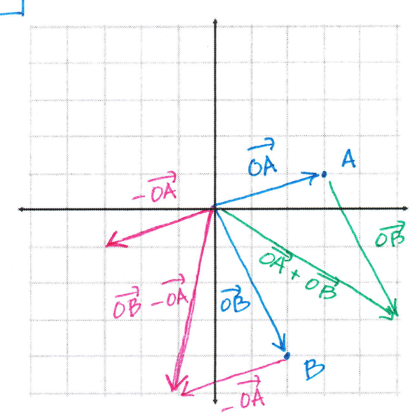


6. Find a vector with magnitude 6 in the direction of $\mathbf{c} = 5\mathbf{i} - 2\mathbf{j}$.

$|\vec{c}| = \sqrt{25+4} = \sqrt{29}$

unit vector $= \left(\frac{5}{\sqrt{29}}\hat{i} - \frac{2}{\sqrt{29}}\hat{j}\right) \xrightarrow{\times 6} \boxed{\frac{30}{\sqrt{29}}\hat{i} - \frac{12}{\sqrt{29}}\hat{j}}$

7. Given A (3, 1) and B (2, –4). Label all vectors that you draw.



a. Graph the position vectors to A and B using the graph at the right.

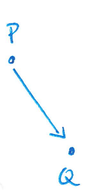
b. Find $\vec{OA}+\vec{OB}$ algebraically and graphically. $\langle 5, -3 \rangle$

c. Find the magnitude of $\vec{OA}+\vec{OB}$. $\sqrt{25+9} = \sqrt{34}$

d. Using the position vectors in part a, graph $\vec{OB}-\vec{OA}$ using the graph at the right. $\vec{OB} + (-\vec{OA})$

e. Find $\vec{OB}-\vec{OA}$ algebraically. $\langle -1, -5 \rangle$

8. $\vec{PQ} = \langle 2, -4 \rangle$



a. Find Q if P = (4, -3).

head-minus-tail

$\langle x-4, y-{-3} \rangle = \langle 2, -4 \rangle$

$x-4 = 2$

$y+3 = -4$

$\boxed{(6, -7)}$

b. Find P if Q = (4, -3).

$\langle 4-x, -3-y \rangle = \langle 2, -4 \rangle$

$4-x = 2$

$-3-y = -4$

$\boxed{(2, 1)}$

9. If **u** = ⟨-1, 5⟩ and **v** = ⟨-10, 3⟩, find the vector projection of **u** onto **v**.

$\text{proj}_v u = \frac{u \cdot v}{|v|^2} v = \left(\frac{10+15}{109}\right)\langle -10, 3\rangle = \boxed{\left(\frac{25}{109}\right)\langle -10, 3\rangle \text{ or } \left\langle -\frac{250}{109}, \frac{75}{109}\right\rangle}$

10. Find the value of a if the vectors ⟨3, 12⟩ and ⟨a, 48⟩ are:

a. parallel

$\langle 3, 12\rangle \xrightarrow{\cdot 4} \langle a, 48\rangle$ $\boxed{a = 12}$

b. orthogonal

dot prod = 0

$3a + 12(48) = 0$

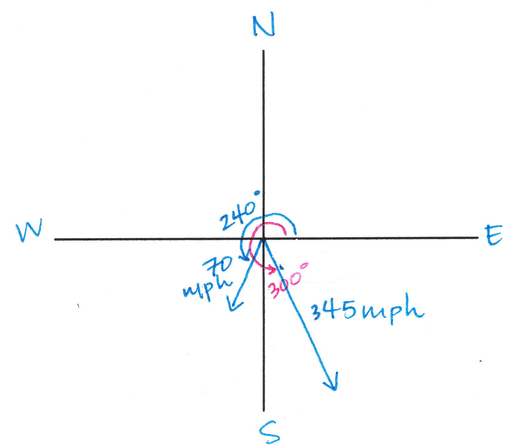
$3a = -12(48)$

$a = -4(48) = \boxed{-192}$

11. Find the angle between the vectors **u** = ⟨-3, 7⟩ and **v** = ⟨2, 5⟩.

$\cos\theta = \frac{u \cdot v}{|u|\,|v|} \rightarrow \cos\theta = \frac{-6+35}{\sqrt{58}\sqrt{29}} \rightarrow \boxed{\theta = 45^\circ}$

12. A ship leaves Honolulu at a bearing of 150 degrees at 345 mph. The wind blows with a bearing of 210 degrees at 70 mph. Draw a vector diagram.



a. Find the component form of the ship and wind velocities.

ship: $\langle 345\cos 300^\circ, 345\sin 300^\circ\rangle$

wind: $\langle 70\cos 240^\circ, 70\sin 240^\circ\rangle$

b. Find the component form of the velocity of the ship after taking into account the wind.

resultant: $\langle \underset{A}{137.500}, \underset{B}{-359.401}\rangle$

c. Find the actual speed and compass bearing of the ship.

$\boxed{\text{actual speed} = 384.805 \text{ mph}}$

direction angle $= \tan^{-1}\left(\frac{B}{A}\right) = -69.064^\circ$

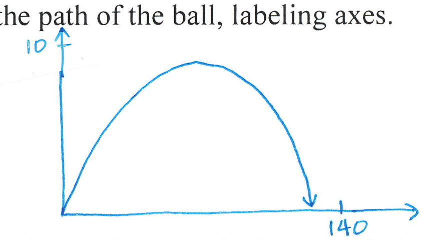
$\boxed{\text{bearing} = 159.064^\circ}$

13. A soccer player kicks a ball from 70 feet away from the goal. The ball is kicked from the ground with an initial velocity of 90 ft/sec at an angle of elevation of 15°.

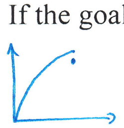
a. Write a set of parametric equations to represent the horizontal and vertical distance of the ball t seconds after it was kicked. Sketch a graph of the path of the ball, labeling axes.

$x = 90\cos 15^\circ t$

$y = -16t^2 + 90\sin 15^\circ t + 0$



b. If the goal is 8 feet tall, will the ball make it into the goal? If not, by how much does the ball miss?



$\boxed{\text{No.}}$

$70 = 90\cos 15^\circ t \rightarrow$ solve for $t \approx 0.805$

$y \approx 8.383$ ft. at that time

$\boxed{\text{Misses by } 0.383 \text{ ft.}}$

c. If the player wanted the ball to enter the goal 7 feet above ground (just above the goalkeeper), at what initial velocity should he have kicked the ball? (Assume the angle of elevation is still 15°).

$7 = -16t^2 + v\sin 15^\circ t$

$70 = v\cos 15^\circ t \rightarrow t = \frac{70}{v\cos 15^\circ}$

$7 = -16\left(\frac{70}{v\cos 15^\circ}\right)^2 + \cancel{v}\sin 15^\circ\left(\frac{70}{\cancel{v}\cos 15^\circ}\right)$

$0.735 = \left(\frac{70}{v\cos 15^\circ}\right)^2$

$\boxed{v \approx 84.543 \text{ ft/sec}}$